SUMMARY  The paper deals with an analytical model of the magnetic field in the air gap of electrical machine with permanent magnets. In this model, the source of radial and tangential magnet fields can be assumed as a system of equivalent current-carrying buses, which thickness depends on permanent magnet properties. Appropriate formulas that describe the problem in question have been derived.

Keywords: Electrical machines, magnetic field, permanent magnets

1. INTRODUCTION

Modern high-energy permanent magnets made of rare earth elements have a magnetic permeability close to the air. For example, Ne-Fe-B magnets $B_r = 1.1$ T and $H_{cB} = 850$ kA/m have a relative differential magnetic permeability

$$
\mu'_u = \frac{B_r}{\mu_0 H_{cB}} = \frac{1.1}{4\pi \cdot 10^{-7} \cdot 0.85 \cdot 10^6} = 1.0298.
$$

Therefore, the calculation of the magnetic field in the core of the electric machine with such magnets can be performed assuming the space occupied by the magnets as an air environment.

We assume further that the operating point on the curve of the return of the magnet is in the second quadrant, and the curve itself in this quadrant is linear.
Microscopic associated molecular currents (Ampère’s currents) in the body of a permanent magnet, compensating each other, are causing the appearance of the surface currents \[1, 2\]. In an ideal version the currents are flowing in an infinitely thin surface layer.

Let us consider a plane-parallel magnetic field of the magnet prism with a rectangular cross-section. It can be assumed that this field is created by two opposing currents of infinitely thin parallel buses with a height \(2b = h_M\) (where \(h_M\) – the height of the magnet in the direction of its magnetization) and at a distance of the width \(b_M\) of the magnet (Fig. 1a).

By arranging the prism in the magnetic air gap of the electric machine and by fixing it to the lower end of the rotor yoke one must take into account two further pairs of buses (as a source of magnetic field), which are mirror images of the original bus arrays of ferromagnetic stator and rotor cores [3].

It will be shown that at the edge of the upper end of the magnet \((x = 0; y = b)\) for an infinitely thin layer of the surface current the tangential component of the magnetic field has an infinite value. However, the experience and the numerical field calculation show a finite value of this field and its sharp decrease due to the displacement of the observation point from the edge of the magnet [4].

Fig. 1. Surface currents of a prismatic magnet with rectangular cross-section, flowing through the infinitely thin buses (a); surface currents at the junction of different polarity magnets (b); field curve of a magnet inductor (c)
When the equivalent bus width is finite, the emissions values of the infinite tangential magnetic field, as shown from corresponding formulas [3, 5], are missing.

This article discusses the calculation of equivalent thickness to the current bus repositories in the air gap of the electric machine and the distribution of the magnetic field generated by them, in particular on the surface of the magnets.

Characteristics depending on $H_x$ along with $H_y$, on the surface of the magnets allow to determine the force exerted by the magnetic field to the magnet, using the tension method [6].

2. MAGNETIC FIELD OF THE RECTANGULAR BUS IN AN OPEN SPACE

Let us consider a bus of a rectangular cross-section with current $I$, located in an open space (Fig. 2). This bus creates the magnetic field, components of which are determined by the following formulas [3, 5]

$$
H_x = \frac{I}{8\pi a b} \left[ (y+b)(\varphi_1 - \varphi_2) - (y-b)(\varphi_3 - \varphi_4) + (x+a)\ln \frac{r_2}{r_4} - (x-a)\ln \frac{r_1}{r_3} \right]; \quad (1)
$$

$$
H_y = -\frac{I}{8\pi a b} \left[ (x+a)(\varphi_2 - \varphi_4) - (x-a)(\varphi_1 - \varphi_3) + (y+b)\ln \frac{r_2}{r_1} - (y-b)\ln \frac{r_4}{r_3} \right], \quad (2)
$$

where $\varphi_k$ – the angle between the positive direction of the axis $x$ and the line segment connecting the observation point $P(x, y)$ with $k$ vertex of the rectangular bus ($k = 1, \ldots, 4$); $r_k$ – the length of these segments.
3. TANGENTIAL MAGNETIC FIELD OF THE JOINT BETWEEN TWO ADJACENT MAGNETS OF OPPOSITE POLARITY ARRANGED IN THE AIR GAP TO THE ROTOR YOKE

Prismatic magnets are mounted on the rotor yoke of the magnetoelectric machine. On the border zone there are the junctions of bipolar magnets (Fig. 1b). The junction as the magnetic field source can be the equivalent bus with the current \(2I\). That bus and its mirror image in the steel rotor yoke (Fig. 3), in accordance with formula (1), will create on line \(y = b\) (on the outer surface of the magnet facing the air gap) the tangential magnetic field

\[
H_x(y = b) = -\frac{I}{2\pi ab} \left[ \frac{-2b \arctg \frac{4ab}{x^2 - a^2 + (2b)^2}}{(x+a)\ln \frac{|x+a|}{\sqrt{(x+a)^2 + (2b)^2}} - (x-a)\ln \frac{|x-a|}{\sqrt{(x-a)^2 + (2b)^2}}} \right] \tag{3}
\]

where \(b = h_M\); \(a\) – equivalent thickness of the bus.

Consideration of the mirror images of the original bus as proposed, presupposes that the magnetic permeability iron cores of the electric machine is equal to infinity\(^1\).

When the thickness of the bus tends to zero, we obtain from (3)

\[
H_x(y = b; x = 0) = -\frac{I}{\pi b} \left[ -1 + \lim_{a \to 0} \frac{a}{2b} \right] = \infty.
\]

Consequently, the end of a thin current-carrying bus will have an infinitely large ejection of the tangential component of the magnetic field. As the experience and numerical calculations refute this conclusion, the problem of determining the thickness of the considered buses should be arisen.

Fig. 3. Bus with a current \(2I\) corresponding to the junction of different polarity magnets and its mirror images in the rotor and stator steel

\(^1\) When the finite values of the magnetic permeability \(\mu\) of the steel, then the value in its physical current \(I\) form decreases to value \(\frac{\mu - 1}{\mu + 1}\). Since in the stator and rotor iron cores \(\mu \gg 1\), the error of the assumption \(\mu = \infty\) is negligible [3].
Another source of the magnetic field, initiated by the joint between the magnets of opposite polarity would be considered i.e. a mirror image of the bus in the steel stator core (Fig. 3). In its own coordinate system, this image will create on the upper end of the source (physical) magnet \((y = -b - 2\delta)\) the tangential magnetic field

\[
H_s(y = -b - 2\delta) = \frac{I}{2\pi ab} \left[ \frac{4a\delta}{x^2 - a^2 + (2\delta)^2} - \frac{4a(b + \delta)}{x^2 - a^2 + (b + 2\delta)^2} \right] \ln \left( \frac{(2\delta)^2 + (x + a)^2}{(2\delta + 2b)^2 + (x + a)^2} \right) + (x + a) \ln \left( \frac{(2\delta)^2 + (x + a)^2}{(2\delta + 2b)^2 + (x + a)^2} \right) - (x - a) \ln \left( \frac{(2\delta)^2 + (x - a)^2}{(2\delta + 2b)^2 + (x - a)^2} \right)
\]

(5)

Summing up the relationship (3) and (5) we obtain an expression as the result of the tangential field of the upper end of the physical magnet

\[
H_t = H_s(y = b) + H_s(y = -b - 2\delta).
\]

(6)

Maximum of the expression (6), corresponding to \(x = 0\), substantially depends on the width \(a\) of the bus

\[
H_{r \text{max}} = H_s(x = 0; y = b) + H_s(x = 0; y = -b - 2\delta),
\]

where

\[
H_s(x = 0; y = b) = \frac{I}{\pi b} \left( 1 - \ln \frac{a}{2b} \right),
\]

(8)

\[
H_s(x = 0; y = -b - 2\delta) = \frac{I}{\pi b} \left( \frac{2\delta}{a} \arctg\frac{a}{2\delta} - 1 + \ln \frac{\delta}{b + \delta} + \frac{1}{2} \ln \left[ 1 + \left( \frac{a}{2\delta} \right)^2 \right] \right).
\]

(9)

Taking into account that \(\frac{a}{2\delta} < 1\), the equation (9) can be simplified by replacing in it the first and second terms by the first two terms of infinite power series [7]

\[
\arctg\frac{a}{2\delta} \approx \frac{a}{2\delta} - \frac{1}{3} \left( \frac{a}{2\delta} \right)^3; \quad \ln \left[ 1 + \left( \frac{a}{2\delta} \right)^2 \right] \approx \left( \frac{a}{2\delta} \right)^2 - \frac{1}{2} \left( \frac{a}{2\delta} \right)^4 \approx \left( \frac{a}{2\delta} \right)^2.
\]

As a result, the formula (9) takes the form

\[
H_s(x = 0; y = -b - 2\delta) = \frac{I}{\pi b} \left[ \frac{1}{24} \left( \frac{a}{\delta} \right)^2 + \ln \frac{\delta}{b + \delta} \right].
\]

(10)
Expression (7) for the maximum value of the resultant tangential field strength, taking into account the formulas (8) and (10), can be written as

\[
H_{r,\text{max}} = H_s(x = 0; y = b) = \frac{I}{\pi b} \left[ 1 - \ln \frac{a}{2b} + \frac{1}{24} \left( \frac{a}{\delta} \right)^2 + \ln \frac{\delta}{b + \delta} \right].
\]  

(11)

The left part of the expression (11) is assumed to be known. Its value can be found from experience or from a data field of numerical calculation. For example, the numerical simulation of the model sample of single-phase magnetoelectric fan motor of the car engine revealed that the ejection of the tangential magnetic flux density at the interface bonded magnets, located opposite the stator tooth, is 0.4 T. We have for this motor following parameters: the height of the magnet \(- h_M = b = 4\) mm; air gap \(- \delta = 0.5\) mm, the coercive force of the magnet by induction \(- H_{cB} = 373.8\) kA/m.

Assuming \(I = H_{cB} h_M\) from the formula (11) we obtain the nonlinear equation for the equivalent thickness of the bus at the junction of different polarity magnets

\[
B_{r,\text{max}} = \frac{\mu_0 H_{cB}}{\pi} \left[ 1 - \ln \frac{a}{2b} + \frac{1}{24} \left( \frac{a}{\delta} \right)^2 + \ln \frac{\delta}{b + \delta} \right].
\]  

(12)

Solving the equation (12) for the motor with the above parameters we obtain \(a = 0.167\) mm.

Additional analysis of the tangential field of magnets on the surface at their junction \((x = 0; y = b)\) indicates that the effect of next two mirror images, closest to the air gap (Fig. 3), is practically absent. Therefore, these tangential field sources (as they are opposite to each other) do not influence the width of the considered equivalent bus.

Figure 4 shows part of the curve of the tangential flux density of the motor \(B_{r} = \mu_0 H_{r}\) built by the equation (6), and a similar dependence obtained by numerical simulations [4].

Now the question arises – is the equivalent thickness of the bus an physical constant value of the magnet or it will be changed by varying the size of the air gap.

Bus current \(I\) is constant. It defines the field of a magnet (Fig. 1c), which in engineering calculations of a magnetoelectric machine one takes it as the constant [8].

The thickness of the bus in view of its low value, does not affect the radial magnetic field, but the effect on the tangential field, as shown above, is significant.

When the location of the magnets joint will be in front of the sufficiently deep and wide slot, the image of stitching buses in the stator will become more distant from the physical joint. But the specific location of the image, unlike the option with a linear boundary steel stator, becomes difficult to install because dealing with nonlinear problem [3]. The magnetic field lines in the region of the slot are not at the bottom thereof, but on wall portions of the slot closest to the air gap. Numerical calculations show that the shift of the junction of the magnets from places with a uniform air gap to a position in front of the stator slot for the magnetoelectric motor [4] will increase the maximum tangential induction from 0.4 to 0.52 T.

The largest increase of the tangential field will obviously be observed for the rotor removed from the stator. In this case, the second term of tangential strength (which determines the demagnetization effect of the stator steel to the tangential field)
can be neglected in the formula (6). As a result, the tangential field should be increased and its maximum flux density, in accordance to formulas (11) (12), will be determined by the expression

\[
B_{t,\text{max}} = \mu_0 H_z(x = 0; y = b) = \frac{\mu_0 H_{\text{at}}}{\pi} \left[ 1 - \ln \frac{a}{2b} \right].
\]  

(13)

Fig. 4. The tangential flux density at the surface of the bonded magnets with pulse ejection at the joints, found by a numerical method for single-phase magnetoelectric motor (curve 1); ejection of the tangential flux density into the joint between the magnets of the same motor, calculated by formula (6) (curve 2)

With regard to the parameters of the considering DC motor (13) we can find \(B_{t,\text{max}} = 0.73\ T\) (Fig. 5, curve 2).

From the numerical calculation\(^2\) of the rotor magnetic field of the motor, removed from the stator (assuming magnetic permeability of all steel parts to be equal to the stator magnetic circuit one \(\mu_0\)), was obtained practically the same value as that from formula (13): \(B_{t,\text{max}} = 0.729\ T\) (Fig. 5, curve 1).

\(^2\) Numerical calculation by the coupling of conformal mappings [9] were performed by A.V. Nikolaev, Ph.D.
A. Afanasyev, A. Nikolaev

This result suggests that the $a$ – the equivalent thickness of the buses with the current, modeling the prismatic surface currents of the magnet, is a constant value of the magnet.

A similar numerical calculation for this rotor, when was excavated out of the stator, but made with Ne-Fe-B magnets ($B_r = 1.1$ T, $H_{cB} = 850$ kA/m) gave to the outer surface of the magnets in their joint $B_{t \text{ max}} = 1.508$ T. Substituting this value of the maximum tangential flux density in formula (13), we obtain the value $a = 0.256$ mm of the magnet. In bonded magnets as described above, we had $a = 0.167$ mm.

Curves of tangential flux density on the surface of the rotor taken out with these magnets are shown in Figure 6.

The increase in the tangential field at the excavated rotor can be explained by natural factors. In this case, because of strong attenuation the radial field, increases the magnetic scalar potential outside of the surface of the magnet, which causes the increase of the tangential magnetic field.

Having the virtual buses in the stator iron core as shown in Figure 3, increases the longitudinal (radial) and weakens the cross (tangential) magnetic field.

If there is a gap between the adjacent edges of the polar zones (butt magnets absent), the maxima of the tangential flux density observed at the edges of different polarity magnets are exactly two times smaller [4].

The physical content of the tangential field magnet joint can be considered as a field of adjacent side scatter of different polarity magnets. Therefore, the proposed method of calculation of this field enables us to quantify the energy efficiency of the use of magnets in an electrical machine.

Fig. 5. The tangential flux density at the surface of the rotor bonded magnets, removed from the stator; curve 1 – numerical calculation, curve 2 – analytical calculation by the formula (3)
Sufficiently reliable calculation of the tangential field of magnets is also necessary to perform the precise calculation of the reactive electromagnetic torque of the machine [6] according to the formula

\[ M = \frac{p l D}{2\mu_0} \int_{-\frac{2\tau}{l}}^{\frac{2\tau}{l}} B_x B_r \, dx \]

where: \( p \) – the number of pole pairs, \( l \) – the estimated machine length, \( D \) – the rotor diameter.

It is shown in Figure 4 that the tangential magnetic flux density created by the junction of the magnets is in the form of a relatively narrow pulse. Therefore, its contribution to the reactive torque is relatively small. The main source of reactive torque is tangential field produced by the teeth-like stator core.

In the load mode, the tangential magnetic field in the air gap will be created by the armature winding as well.

4. RADIAL MAGNETIC FIELD IN THE AIR GAP CAUSED BY EQUIVALENT MAGNETIC BUSSES

Equation (2) allows to obtain an expression for the radial component of the magnetic flux density in the air gap caused by the equivalent magnetic bus with current
\[ I = 2HcB h_M \] of a neighboring magnets joint and an infinite number of its mirror images of the steel rotor and the stator. This expression applied to the observation points on the circumference of the stator bore \((y = b + \delta)\) will look like

\[
B_y(x, y = b + \delta) = \sum_{n=1}^{\infty} \left\{ \frac{2b(x+a)}{(x+a)^2 + [2bn + \delta(2n-1)][2b(n-1)] + \delta(2n-1)} - \frac{2b(x-a)}{(x-a)^2 + [2bn + \delta(2n-1)][2b(n-1)] + \delta(2n-1)} \right\} + [2b(n-1) + \delta(2n-1)] \ln \frac{(x+a)^2 + [2bn + \delta(2n-1)]^2}{(x-a)^2 + [2bn + \delta(2n-1)]^2} - [2b(n-1) + \delta(2n-1)] \ln \frac{(x+a)^2 + [2bn + \delta(2n-1)]^2}{(x-a)^2 + [2bn + \delta(2n-1)]^2}.
\]  
\[ (14) \]

Two similar buses, with the current \(I\) of opposite sign and located to the left and to the rights of the original buses at a distance of pole pitch \(\tau\), will create their radial magnetic fields that are added to the field (14). These fields may also be calculated by the formula (14), taking instead the variable \(x\), respectively, the variables \((x + \tau)\) and \((x - \tau)\). To account the effect of polar zones of adjacent periods to a given period it can be for the found field to add bus fields, with the corresponding shifts on the \(\pm \pi\), \((s = 2, 3\ldots)\).

In a case of toothing the air gap should be on the first conformally mapped on the upper complex half-plane, then on the endless bandwidth in the other complex plane. In this bandwidth equivalent current buses will have a configuration other than rectangular. Therefore, they should be replaced by a set of linear currents [11], whose magnetic fields and their mirror images in the stator and rotor steel can be easily found.

It is possible a relatively simple approximate method of accounting tooth-like stator. For this the magnetic field is calculated by the formula (14), performed precisely by finite function \(k(x)\) as a serving factor in this formula

\[
B_y(x) = k(x) B_y(x, y = b + \delta),
\]

where

\[
k(x) = \begin{cases} 
    k_j(x) = 1 - A + A \cos(x - x_{y,j} \frac{2\pi}{b_n}), & x_{y,j} \leq x \leq x_{y,j} + b_n; \\
    1, & x_{y(j-1) + b_n} \leq x \leq x_{y,j},
\end{cases}
\]

\[ x_{y,j} = x_{y,j} - (\sigma - 1)/2, \quad x_{y,j} = \text{coordinate of the left edge of the j-th slot}; \quad b_n = b_n \sigma; \]
\[ \sigma > 1 \text{ – magnification equivalent to the width of the slot zone [10];} \]
\[ A = B_{\delta \max} \beta, B_{\delta \max} \text{ – the maximum value of the magnetic flux density in the tooth bordering the slot; the coefficient } \beta \text{ depends on the ratio of } \delta/b_n [10]. \]
The function (16) can take into account the rotational motion of the rotor, if in it to set the angular coordinate of the rotor $\vartheta = \omega t$

$$k(x, \vartheta) = k(x - \vartheta) = k(x - \omega t).$$  \hfill (17)

Figure 7 shows the curves of magnetic flux density (analytical and numerical) on the circumference of the stator bore of a single-phase magnetoelectric motor [4], in which one-third of the spatial period is not a magnet — for the purpose of fixing the prelaunch starting position of the rotor relative to the stator coils with six claw and six narrow teeth without coils. Some disagreement between them can be attributed to the neglect of magnetic saturation in the analytical method.

![Figure 7. Magnetic flux density at the bore surface of the stator core of a single-phase magnetoelectric motor: curve 1 is calculated according to the formulas (14)...(16) with the number of terms in (14) equal to 10 000 and curve 2 was obtained numerically](image)

5. CONCLUSIONS

1. Plane-parallel magnetic field generated by the magnet with rectangular prismatic cross-section, located in the air gap at the rotor yoke of the electrical machine, can be modeled by the magnetic field of the equivalent buses with counter currents $I = H_{c,M} h_M$ and their mirror images in the rotor and stator steel. Spacing between bus axes is equal to the magnet width and a height of the bus coincides with the height of the magnet.
2. Equivalent thickness of buses, which is a parameter of the magnet material, depends on the maximum tangential flux density on the surface of the magnet, determined by numerical simulations or obtained from an experience.

3. When you remove a rotor with permanent magnets out of the stator, the maximum tangential flux density on the surface of the magnets will be the greatest.

4. The system under consideration of the equivalent current-carrying buses allows to take into account the impact on the teeth edges of the stator magnetic field in the air gap.

5. The proposed method of determining the tangential field of the joint for different polarity magnets can be considered as a method of calculating the lateral scattering of magnets.

LITERATURE


Manuscript submitted 11.07.2013

MATEMATYCZNY MODEL POLA MAGNESU TRWAŁEGO W SZCZELINIE POWIETRZNEJ MASZYNY ELEKTRYCZNEJ

Alexander AFANASYEV, Alexej NIKOLAEV

STRESZCZENIE W artykule zaproponowano oryginalny analityczny model pola magnetycznego w szczelinie powietrznej maszyny elektrycznej z magnesami trwałymi. W modelu tym źródłami składowych: radialnej i tangensjonalnej pola magnetycznego są ekwiwalentne układy przewodów z prądem, których grubość zależy od własności materiałowych magnesu trwałego. Wyprowadzono wzory i zależności matematyczne opisujące rozważany model pola.

Słowa kluczowe: maszyny elektryczne, pole magnetyczne, magnesy trwałe